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Transformation of I – Function and H-Function for Rational Parameters

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ABSTRACT: In this paper we have given the special cases of I-function when the coefficients of 's' in respective of gamma functions in the integrands are rational numbers. The formulas investigated in this paper will unify the higher cadre of generalized hypergeometric functions.

I. INTRODUCTION

(i) The G- Function

Meijer's G – function provides an interpretation of the symbol ${}_pF_q$, when p> q+1. This interpretation is incomplete agreement with the given by MacRobert 's E – function. The G- function was defined by Meijer [5] in Mellin – Barnes type integrals as follows:

$$G_{p,q}^{m,n}\left[x\Big|{}_{b_1, b_2, \dots\dots\dots\dots b_q}^{a_1\,, a_2, \dots\dots\dots\dots a_p}\right] =$$

$$\frac{1}{2\pi i}\int_L \frac{\prod_{j=1}^{m}\Gamma\,(b_j-\xi)\,\prod_{j=1}^{n}\Gamma\,(1-a_j+\xi)}{\prod_{j=m+1}^{q_1}\Gamma\,(1-b_j+\xi)\,\prod_{j=1}^{p_1}\Gamma\,(a_j-\xi)}\, x^{\xi}\ d\xi \qquad \dots\dots..(1.1)$$

Where an empty product is interpreted as 1, $0 \le m \le q, 0 \le n \le p$ and the parameters are such that no pole of $\Gamma(b_j-\xi)\,(j = 1, 2 \dots . m)$ coincides with any pole of Γ (1- a_j + ξ) (j = 1 , 2 ….n). these assumptions will be retained throughout. There are three different paths L of integration.

(i) L runs from - $i\infty$ to $i\infty$ so that all poles of Γ (b_j - ξ) (j= 1,2,…………m), are to the right, and all the poles of Γ (1- aj + ξ) (j = 1,2,………n) to the left of L. the integral converges if p+q <2 (m+n) and |arg x | < (m+n - $\frac{1}{2}p - \frac{1}{2}q$) π .

(ii) L is a loop starting and ending at + ∞ and encircling all poles of Γ (b_j - ξ) (j=1,2,……m) once in the negative direction, but none of the pole of Γ (1- aj + ξ) (j= 1,2 ,…..n). the integral converges if q≥ 1 and either p< q or p = q and |x| < 1.

(iii) L is a loop starting and ending at -∞ and encircling all poles of Γ (1-a_j + ξ) (j=1,2,……n) once in the positive direction, but none of the poles of Γ (b_j - ξ) (j=1,2,………….m). The integral converges if p ≥ 1and p > q or p = q and |x| >1.

(ii) The H- Function

The H- function occurs in the study of the solutions of certain function equations considered by Bochner [1] and Chandrasekharan and Narsimhan [2].

In an attempt to unity and extend the existing results on symmetrical Fourier kernels, Fox [3] has defined the H-function in terms of a general Mellin – Barne 's type integral. He also investigated the most general Fourier Kernel associated with the H- function and obtained the asymptotic expansions of the kernel for large values of the argument, by following his earlier method [4].

The H- function is defined in terms Mellin Barne's type integral as follows:

$$H_{p,q}^{m,n}\left[x\,\Big|\,{}_{(b_j\ ,\ \beta_j)_{1,q}}^{(a_j\ ,\ \alpha_j)_{1,p}}\right] = \frac{1}{2\pi i}\int_L\ \emptyset\ (\xi)\ x^{\xi}\ d\xi \qquad \dots(1.2)$$

Where x^{ξ} = exp [ξ log |x| + i arg x] ...(1.3)

In which log $|x|$ represents the natural logarithm of $|x|$ and arg x is not necessarily the principal value.

$$\emptyset\,(\xi) = \frac{\prod_{j=1}^{m} \Gamma\left(b_j - \beta_j\, \xi\right) \quad \prod_{j=1}^{n} \quad \Gamma\left(1 - a_j + \alpha_j\, \xi\right)}{\prod_{j=m+1}^{q_i} \Gamma\left(1 - b_j + \beta_j\, \xi\right) \ \prod_{j=n+1}^{p_i} \Gamma\left(a_j - \alpha_j\, \xi\right)} \quad \ldots (1.4)$$

Where an empty product is interpreted as unity and m, n, p, q are non negative integers such that $0 \leq n \leq p, 0 \leq m \leq q, \alpha_j$(j = 1,2,…….p) β_j (j=1,2,……q) are positive numbers. a_j(j=1,2,…..p) b_j(j=1,2,……..q) are complex number such that none of the points $\xi = (b_h + \upsilon)\ /\ \beta_h$ and $\xi = (a_k - \eta - 1)\ /\ \alpha_k$ coincide with another, i.e. ,

$$\alpha_k(b_h + \upsilon\,) \ \neq\ \beta_h\,(a_k - \eta - 1)\ for\ h = 1,2,\ldots m\ , \qquad k = 1,2,\ldots n, \upsilon = \eta = 0,1,2,\ldots,$$

The contour L runs from $-\,i\infty\ to + i\infty$ such that the poles of $\Gamma\,(b_h - \beta_h\,\xi)$, h = 1,2,…m, lie to the right of L and poles of $\Gamma\,(1 - a_k + \alpha_k\,\xi), k = 1,2,\ldots n$ lie to the left of L.

(iii) The I – Function

The I – function defined by Saxena, V.P. is the latest and most general form of hyper geometric functions. This function emerged by itself while solving a class of dual integral equations involving Fox ‘s H –function as kernels [6]. The I – function is defined in terms of following Mellin – Barnes type integral,

$$I_{p_i, q_i\,:\,r}^{m,n}\left[x\ \middle|\ \begin{matrix}(a_j\,,\ \alpha_j)_{1,n}\,;\ (a_{ji}\,, \alpha_{ji}\,)_{n+1\,,pi}\\ (b_{j,}\ \beta_j\,)_{1,m}\,;\ (b_{ji}\,, \beta_{ji})_{m+1,qi}\end{matrix}\right] = \frac{1}{2\pi i}\int_L \psi\,(\xi)\ x^{\xi}\ d\xi - (1.4)$$

Where x^{ξ} = exp [ξ log $|x| + i \arg x$] ………………………………(1.5)

In which log $|x|$ represent the natural logarithm of $|x|$ and arg x is not necessarily the principal value.

Here

$$\Psi(\xi) = \frac{\prod_{j=1}^{m}\ \Gamma(b_j - \beta_j\,\xi)\ \ \prod_{j=1}^{n}\ \Gamma(1 - a_j + \alpha_j\,\xi)}{\sum_{i=1}^{r}\ \ \prod_{j=m+1}^{q_i}\ \Gamma(1 - b_{ji} + \beta_{ji}\,\xi)\ \prod_{j=n+1}^{p_i}\ \Gamma(a_{ji} - \alpha_{ji}\ \xi)} \ldots\ldots\ldots (1.6)$$

Where an empty product interpreted as unity and, m, n, p_i, q_i , r, are non – negative integers such that $0 \leq n \leq p_i\,, 0\ \leq m\ \leq\ q_i\,, \alpha_j$ (j=1,2,……….n), β_j (j=1,2,…………m), α_{ji} (j=n+1,n+2,……….p_i , i = 1,2,…………r), β_{ji} (j=m+1, m+2, …..q_i , i= 1,2,……….r) are positive numbers and a_j (j=1,2,……..n), b_j (j=1,2,……m) , a_{ji} (j=n+1,n+2,….p_i, i=1,2,….r) , b_{ji} (j=m+1,m+2,………..q_i , i=1,2,..r) are complex numbers such that none of the points $\xi = (b_h + \upsilon)\ /\ \beta_h\ and\ \xi\ = (a_k - \eta - 1)\ /\ \alpha_k$ coincide with each other , i.e.

$$\alpha_k(b_h + \upsilon)\ \neq\ b_h\,(a_k - \eta - 1), for\ h = 1,2,\ldots\ldots m, k\ = 1,2,\ldots.\,.\,n, \upsilon\ =\ \eta\ = 0,1,2\ldots\ldots\ldots\ldots.,$$

L is a contour runs from $\sigma - i\infty\ to\ \sigma + i\infty\ (\sigma\ is\ real), in\ the\ complex$ ξ – span such that the poles of $\Gamma\ (b_j - \beta_j\,\xi)$ lie to the right hand side of L and the poles of $\Gamma(\ 1 - a_j + \alpha_j\,\xi\ )$ lie to the left hand side of L.

II. TRANSFORM H TO G

We have

$$H_{p,q}^{m,n}[z]\ = H_{p,q}^{m,n}\left[z\ \middle|\begin{matrix}(a_1, \alpha_1\,)\ldots\ldots(a_p, \alpha_p)\\ (b_1, \beta_1)\ldots\ldots(b_q, \beta_q)\end{matrix}\right]$$

$$= \frac{1}{2\pi i}\int_L t\,(s) z^s ds \qquad \ldots(2.1)$$

where

$$t\,(s) \quad = \frac{\prod_{j=1}^{m}\Gamma\left(b_j - \beta_j s\right)\ \prod_{j=1}^{n}\Gamma\left(1 - a_j + \alpha_j s\right)}{\prod_{j=m+1}^{q}\Gamma\left(1 - b_j + \beta_j s\right)\ \prod_{j=n+1}^{p}\Gamma\left(a_j - \alpha_j s\right)} \qquad \ldots(2.2)$$

Now,

$$H_{p,q}^{m,n}\ [z] = \frac{1}{2\pi i}\int_L \frac{\prod_{j=1}^{m}\Gamma\left(b_j - \beta_j s\right)\ \prod_{j=1}^{n}\Gamma\left(1 - a_j + \alpha_j s\right)}{\prod_{j=m+1}^{q}\Gamma\left(1 - b_j - \beta_j s\right)\ \prod_{j=n+1}^{p}\Gamma\left(a_j - \alpha_j s\right)}\ z^s ds \qquad \ldots(2.3)$$

Put $\alpha_j = M_j$, $\beta_j = N_j$ in (2.3), we get

$$= \frac{1}{2\pi i}\int_L \frac{\prod_{j=1}^{m}\Gamma(b_j - N_j s)\ \prod_{j=1}^{n}\Gamma(1 - a_j + M_j s)}{\prod_{j=m+1}^{q}\Gamma(1 - b_j + N_j s)\ \prod_{j=n+1}^{n}\Gamma(a_j - M_j s)}\ Z^s\ ds$$

$$=\frac{1}{2\pi i}\int_L \frac{\prod_{j=1}^{m}\Gamma\left(N_j\left(\frac{b_j}{N_j}-s\right)\right)\prod_{j=1}^{n}\Gamma\left(M_j\left(\frac{1-q_j}{M_j}+s\right)\right)}{\prod_{j=m+1}^{q}\Gamma\left(N_j\left(\frac{1-b_j}{N_j}+s\right)\right)\prod_{j=n+1}^{p}\Gamma\left(M_j\left(\frac{q_j}{M_j}-s\right)\right)} z^s ds \qquad ...(2.4)$$

Put $\frac{b_j}{N_j}$ -s = Z_j, $\frac{1-a_j}{M_j}+s=\gamma_j$, $\frac{1-b_j}{N_j}+\ s=Y_j$, $\frac{a_j}{M_j}-s=X_j$ in (2.4),

we get

$$=\frac{1}{2\pi i}\int_L \frac{\prod_{j=1}^{m}\Gamma(N_jZ_j)\prod_{j=1}^{n}\Gamma(M_jY_j)}{\prod_{j=m+1}^{q}\Gamma(N_jY_j)\prod_{j=n+1}^{p}\Gamma(M_jX_j)} z^s ds \qquad ...(2.5)$$

By using formula,[7 ,p-66]

$$\Gamma(mZ)=(2\pi)^{\frac{1-m}{2}}\ m^{z-1/2}\prod_{j=1}^{m}\left(Z+\frac{j-1}{m}\right),$$

$$Z\neq 0,-1/m, 2/m \ldots\ldots m=1,2,3 \qquad ...(2.6)$$

Then,

$$\prod_{j=1}^{m}\Gamma(N_jZ_j)=\Gamma(N_1Z_1)\Gamma(N_2Z_2)\Gamma(N_3Z_3)\ldots\ldots\ldots\Gamma(N_mZ_m)$$

$$=\left[(2\pi)^{\frac{1-N_1}{2}}(N_1)^{Z_1-1/2}\prod_{j=1}^{N_1}\Gamma\left(Z_1+\frac{j-1}{N_1}\right)\right]$$

$$\left[(2\pi)^{\frac{1-N_2}{2}}(N_2)^{Z_2-1/2}\prod_{j=1}^{N_2}\Gamma\left(Z_2+\frac{j-1}{N_2}\right)\right]$$

$$\left[(2\pi)^{\frac{1-N_3}{2}}(N_3)^{Z_3-1/2}\prod_{j=1}^{N_3}\Gamma\left(Z_3+\frac{j-1}{N_3}\right)\right]\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$\left[(2\pi)^{\frac{1-N_m}{2}}(N_m)^{Z_m-1/2}\prod_{j=1}^{N_m}\Gamma\left(Z_m+\frac{j-1}{N_m}\right)\right]$$

$$=(2\pi)^{(1/2+1/2+1/2\ldots\ldots\text{mtimes})}.(2\pi)^{-\left[\frac{N_1+N_2+N_3+\cdots\ldots+N_m}{2}\right]}.\quad (N_1.N_2.N_3\ldots..N_m)^{-1/2}$$

$$\prod_{j=1}^{m}(N_j)^{Z_j}.\prod_{j=1}^{N_1}\Gamma\left(\frac{b_1}{N_1}-s+\frac{j-1}{N_1}\right).\prod_{j=1}^{N_2}\Gamma\left(\frac{b_2}{N_2}-s+\frac{j-1}{N_2}\right).$$

$$\prod_{j=1}^{N_3}\Gamma\left(\frac{b_3}{N_3}-s+\frac{j-1}{N_3}\right)\ldots\prod_{j=1}^{N_m}\Gamma\left(\frac{b_m}{N_m}-s+\frac{j-1}{N_m}\right)$$

$$=(2\pi)^{m/2}.(2\pi)^{-\left[\frac{N_1+N_2+N_3+\cdots+N_m}{2}\right]}.(N_1.N_2\ldots N_m)^{-1/2}.$$

$$N_1^{\left(\frac{b_1}{N_1}-s\right)}.N_2^{\left(\frac{b_2}{N_2}-s\right)}.N_3^{\left(\frac{b_3}{N_3}-s\right)}\ldots N_m^{\left(\frac{b_m}{N_m}-s\right)}.\prod_{j=1}^{N_1}\Gamma\left(\frac{b_1+j-1}{N_1}-s\right).$$

$$\prod_{j=1}^{N_2}\Gamma\left(\frac{b_2+j-1}{N_2}-s\right).\prod_{j=1}^{N_3}\Gamma\left(\frac{b_3+j-1}{N_3}-s\right)\ldots..\prod_{j=1}^{N_m}\left(\frac{b_m+j-1}{N_m}-s\right).$$

$$=(2\pi)^{m/2}.(2\pi)^{\frac{-(N_1+N_2+N_3+\ldots..+N_m)}{2}}.(N_1.N_2.N_3\ldots N_m)^{-1/2}(N_1.N_2.N_3.\ldots.N_m)^{-s}$$

$$N_1^{\frac{b_1}{N_1}}.N_2^{\frac{b_2}{N_2}}N_3^{\frac{b_3}{N_3}}\ldots\ldots\ldots\ldots N_m^{\frac{b_m}{N_m}}.\prod_{j=1}^{N_1}\Gamma\left(\frac{b_1+j-1}{N_1}-s\right)$$

$$\prod_{j=1}^{N_2}\Gamma\left(\frac{b_2+j-1}{N_2}-s\right).\prod_{j=1}^{N_3}\Gamma\left(\frac{b_3+j-1}{N_3}-s\right)\ldots..\prod_{j=1}^{N_m}\left(\frac{b_m+j-1}{N_m}-s\right)$$

$$=(2\pi)^{m/2}.(2\pi)^{\frac{-(N_1+N_2+N_3+\ldots..+N_m)}{2}}.(N_1.N_2.N_3\ldots N_m)^{-1/2}$$

$$\prod_{j=1}^{m}(N_j)^{-s}\ N_j^{\frac{b_j}{N_j}}\prod_{i=1}^{N}\Gamma(\beta_i-s) \qquad \ldots(2.7)$$

Where $\beta_i=\frac{b_i+j-1}{N_i}$

$N=N_1+N_2+N_3+\cdots\ldots+N_m, j=1\text{to}N.$

$$\prod_{j=1}^{n}\Gamma(M_j\gamma_j)=\Gamma(M_1\gamma_1)\Gamma(M_2\gamma_2).\Gamma(M_3\gamma_3)\ldots..\Gamma(M_n\gamma_n)$$

$$=\left[(2\pi)^{\frac{1-M_1}{2}}(M_1)^{\gamma_1-1/2}\prod_{j=1}^{M_1}\Gamma\left(\gamma_1+\frac{j-1}{M_1}\right)\right].$$

$$\left[(2\pi)^{\frac{1-M_2}{2}}M_2^{\gamma_2-1/2}\prod_{j=1}^{M_2}\Gamma\left(\gamma_2+\frac{j-1}{M_2}\right)\right].$$

$$\left[(2\pi)^{\frac{1-M_3}{2}}(M_3)^{\gamma_3-1/2}\prod_{j=1}^{M_3}\left(\gamma_3+\frac{j-1}{M_3}\right)\right]\ldots\ldots\ldots$$

$$\left[(2\pi)^{\frac{1-M_n}{2}}(M_n)^{\gamma_n-1/2}\prod_{j=1}^{M_n}\left(\gamma_n+\frac{j-1}{M_n}\right)\right]$$

$= (2\pi)^{(1/2+1/2+1/2 ......ntimes)} . (2\pi)^{-\left[\frac{M_1+M_2+M_3+\cdots+M_n}{2}\right]} (M_1.M_2.M_3 .....M_n)^{-1/2}$
$\left[M_1^{\frac{1-a_1}{M_1}}.M_2^{\frac{1-a_2}{M_2}} ....M_n^{\frac{1-a_n}{M_n}}\right] (M_1.M_2.M_3 .....M_n)^{+s}. \prod_{j=1}^{M_1}\Gamma\left(\frac{1-a_1}{M_1}+s+\frac{j-1}{M_1}\right). \prod_{j=1}^{M_2}\Gamma\left(\frac{1-a_2}{M_2}+s+\frac{j-1}{M_2}\right).$
$\prod_{j=1}^{M_3}\Gamma\left(\frac{1-a_3}{M_3}+s+\frac{j-1}{M_3}\right)....... \prod_{j=1}^{M_n}\Gamma\left(\frac{1-a_n}{M_n}+s+\frac{j-1}{M_n}\right).$

$= (2\pi)^{n/2} (2\pi)^{-\left[\frac{M_1+M_2+M_3+\cdots+M_n}{2}\right]}(M_1.M_2 ... M_n)^{-1/2} \prod_{j=1}^{n}(M_j)^{\frac{1-a_j}{M_j}} (M_j)^s.$
$\prod_{j=1}^{M_1}\Gamma\left(\frac{j-a_1}{M_1}+s\right).\prod_{j=1}^{M_2}\Gamma\left(\frac{j-a_2}{M_2}+s\right).\prod_{j=1}^{M3}\Gamma\left(\frac{j-a_3}{M_3}+s\right)......\prod_{j=1}^{M_n}\Gamma\left(\frac{j-a_n}{M_n}+s\right)$

$= (2\pi)^{n/2} (2\pi)^{-\frac{[M_1+M_2+\cdots M_n]}{2}} . (M_1.M_2 ... M_n)^{-1/2} \prod_{j=1}^{n}(M_j)^{\frac{1-a_j}{M_j}} (M_j)^s$
$\prod_{i=1}^{M}\Gamma(\alpha_i+s)$ ... (2.8)

Where $\alpha_i = \frac{j-a_i}{M_i}+s$

$M = M_1 + M_2 + M_3 + \cdots. +M_n$

$\prod_{j=M+1}^{q}\Gamma(N_jY_j)=\Gamma(N_{m+1}Y_{m+1}).\Gamma(N_{m+2}Y_{m+2}).\Gamma(N_{m+3}Y_{m+3}) ... .\Gamma(N_qY_q)$

$= \left[(2\pi)^{\frac{1-N_{m+1}}{2}} (N_{m+1})^{Y_{m+1}-1/2} \prod_{j=1}^{N_{m+1}}\Gamma\left(Y_{m+1}+\frac{j-1}{N_{m+1}}\right)\right].$
$\left[(2\pi)^{\frac{1-N_{m+2}}{2}} (N_{m+2})^{Y_{m+2}-1/2} \prod_{j=1}^{N_{m+2}}\Gamma\left(Y_{m+2}+\frac{j-1}{N_{m+2}}\right)\right].$
.
.
$\left[(2\pi)^{\frac{1-N_q}{2}} (N_q)^{Y_q-1/2} \prod_{j=1}^{N_q}\Gamma\left(Y_q+\frac{j-1}{N_q}\right)\right]$

$= (2\pi)^{\frac{q-m}{2}} . (2\pi)^{-\frac{[N_{m+1}+N_{m+2}+\cdots+N_q]}{2}}.\prod_{j=m+1}^{q}(N_j)^{\frac{1-b_j}{N_j}} (N_j)^s \prod_{j=m+1}^{q}(N_j)^{-1/2}$
$\prod_{j=1}^{N_{m+1}}\Gamma\left(\frac{1-b_{m+1}}{N_{m+1}}+s+\frac{j-1}{N_{m+1}}\right).\prod_{j=1}^{N_{m+2}}\Gamma\left(\frac{1-b_{m+2}}{N_{m+2}}+s+\frac{j-1}{N_{m+2}}\right).$
$\prod_{j=1}^{N_{m+3}}\Gamma\left(\frac{1-b_{m+3}}{N_{m+3}}+s+\frac{j-1}{N_{m+3}}\right)............... \prod_{j=1}^{N_q}\Gamma\left(\frac{1-b_q}{N_q}+s+\frac{j-1}{N_q}\right)$

$= (2\pi)^{\frac{q-m}{2}} (2\pi)^{-\frac{[N_{m+1}+N_{m+2}+\cdots+N_q]}{2}}.\prod_{j=m+1}^{q}(N_j)^{\frac{1-b_j}{N_j}} (N_j)^s \prod_{j=m+1}^{q}(N_j)^{-1/2}.$
$\prod_{j=1}^{N_{m+1}}\Gamma\left(\frac{j-b_{m+1}}{N_{m+1}}+s\right) \prod_{j=1}^{N_{m+2}}\Gamma\left(\frac{j-b_{m+2}}{N_{m+2}}+s\right).........\prod_{j=1}^{N_q}\Gamma\left(\frac{j-b_q}{N_q}+s\right)$

$= (2\pi)^{\frac{q-m}{2}} (2\pi)^{-\frac{[N_{m+1}+N_{m+2}+\cdots+N_q]}{2}} . \prod_{j=m+1}^{q}(N_j)^{\frac{1-b_j}{N_q}} (N_j)^s \prod_{j=m+1}^{q}(N_j)^{-1/2} \prod_{i=m+1}^{Q}\Gamma\left(\frac{j-b_i}{N_i}+s\right)$

$= (2\pi)^{\frac{q-m}{2}} (2\pi)^{-\frac{[N_{m+1}+N_{m+2}+\cdots N_q]}{2}}. \prod_{j=m+1}^{q}(N_j)^{\frac{1-b_j}{N_a}} (N_j)^s \prod_{j=m+1}^{q}(N_j)^{-1/2}$
$\prod_{i=m+1}^{Q}\Gamma(\lambda_i+s)$ ... (2.9)

Where

$Q = N_{m+1} + N_{m+1} + N_{m+3} + \cdots + N_q$

$\lambda_i = \Gamma\left(\frac{j-b_i}{N_i}+s\right)$

$\prod_{j=n+1}^{P}\Gamma(M_jX_j) = \Gamma(M_{n+1}X_{n+1})\,\Gamma(M_{n+2}X_{n+2})\,\Gamma(M_{n+3}X_{n+3}) .....\Gamma(M_PX_P)$

$= \left[(2\pi)^{\frac{1-M_{n+1}}{2}} (M_{n+1})^{X_{n+1}-1/2} \prod_{j=1}^{M_{n+1}}\Gamma\left(X_{n+1}+\frac{j-1}{M_{n+1}}\right)\right].$
$\left[(2\pi)^{\frac{1-M_{n+2}}{2}} (M_{n+2})^{X_{n+2}-1/2} \prod_{j=1}^{M_{n+2}}\Gamma\left(X_{n+2}+\frac{j-1}{M_{n+2}}\right)\right].$
$\left[(2\pi)^{\frac{1-M_{n+3}}{2}} (M_{n+3})^{X_{n+3}-1/2} \prod_{j=1}^{M_{n+3}}\Gamma\left(X_{n+3}+\frac{j-1}{M_{n+3}}\right)\right].........$
$.......\left[(2\pi)^{\frac{1-M_P}{2}} (M_P)^{X_P-1/2} \prod_{j=1}^{M_P}\Gamma\left(X_P+\frac{j-1}{M_P}\right)\right]$

$= (2\pi)^{\frac{p-n}{2}} (2\pi)^{-\frac{[M_{n+1}+M_{n+2}+\cdots+M_p]}{2}} \prod_{j=n+1}^{p}(M_j)^{\frac{a_j}{M_j}} (M_j)^{-s} \prod_{j=n+1}^{R}(M_j)^{-1/2} \prod_{i=n+1}^{p}\Gamma\left(\frac{a_i+j-1}{M_i}-s\right)$

Where $R = M_{n+1} + M_{n+2} + \cdots + M_p$

$$\delta_i = \frac{a_i + j - 1}{M_i}$$

$$= (2\pi)^{\frac{p-n}{2}} (2\pi)^{-\frac{[M_{n+1}+M_{n+2}+\cdots+M_p]}{2}} . \prod_{j=n+1}^{p} (m_j)^{\frac{a_j}{M_j}} (M_j)^{-s} \quad \prod_{j=m+1}^{p} (M_j)^{-1/2} \prod_{i=n+1}^{R} \Gamma(\delta_i - s) \quad ...(2.10)$$

Putting in equation (2.3) we get

$$H_{p,q}^{m,n}[z] = \int_L (2\pi)^{m/2} (2\pi)^{-\frac{[N_1+N_2+....+N_m]}{2}} \times \prod_{j=1}^{m} (N_j)^{-1/2} . \prod_{j=1}^{m} (N_j)^{\frac{b_j}{N_j}} \quad (2\pi)^{n/2} (2\pi)^{-\frac{[M_1+M_2+...+M_n]}{2}} \prod_{j=1}^{n} (M_j)^{-1/2}$$

$$\times \frac{\prod_{j=1}^{n} (M_j)^{\frac{1-a_j}{n_j}} \prod_{i=1}^{N} \Gamma(\beta_i - s) \prod_{i=1}^{M} \Gamma(\alpha_i + s) \prod_{j=1}^{m} (N_j)^{-s} \prod_{j=1}^{n} (M_j)^{s}}{(2\pi)^{\frac{q-m}{2}} (2\pi)^{-\frac{[N_{m+1}+N_{m+2}+\cdots+N_q]}{2}} \prod_{j=m+1}^{q} (N_j)^{\frac{1-b_j}{N_j}} \prod_{j=m+1}^{q} (N_j)^{-1/2} (2\pi)^{\frac{p-n}{2}}}$$

$$\times \frac{1}{(2\pi)^{-\frac{[M_{n+1}+M_{n+2}+..+M_p]}{2}} \prod_{j=m+1}^{p} (M_j)^{\frac{a_j}{M_j}} \prod_{j=m+1}^{q} (M_j)^{-1/2} \prod_{i=m+1}^{Q} \Gamma(\lambda_i + s)}$$

$$\times \frac{1}{\prod_{i=n+1}^{R} \Gamma(\delta_i - s) \prod_{j=m+1}^{q} (N_j)^{s} \prod_{j=n+1}^{p} (M_j)^{-s}} z^s ds$$

$$= (2\pi)^{\frac{m+n-q+m-p+n}{2}} \quad (2\pi)^{-\frac{[(N_1+N_2+...+N_m)+(M_1+M_2+\cdots M_n)+(N_{m+1}+N_{m+2}+\cdots+N_q)+(M_{n+1}+M_{n+2}+\cdots+M_p)]}{2}}$$

$$\times \frac{\prod_{j=1}^{m} (N_j)^{-1/2} \prod_{j=1}^{m} (N_j)^{\frac{b_j}{N_j}} \prod_{j=1}^{n} (M_j)^{-1/2} \prod_{j=1}^{n} (M_j)^{\frac{1-a_j}{n_j}} .}{\prod_{j=m+1}^{q} (N_j)^{\frac{1-b_j}{N_j}} \prod_{j=m+1}^{q} (N_j)^{-1/2} \prod_{j=m+1}^{p} (M_j)^{\frac{a_j}{M_j}} \prod_{j=m+1}^{q} (M_j)^{-1/2}}$$

$$\times \int_L \frac{\prod_{i=1}^{N} \Gamma(\beta_i - s) \prod_{i=1}^{M} \Gamma(\alpha_i + s) \prod_{j=1}^{m} (N_j)^{-s} \prod_{j=1}^{n} (M_j)^{s}}{\prod_{i=m+1}^{Q} \Gamma(\lambda_i + s) \prod_{i=n+1}^{R} \Gamma(\delta_i - s) \prod_{j=m+1}^{q} (N_j)^{s} \prod_{j=n+1}^{p} (M_j)^{-s}} z^s ds$$

Where $\eta = \sum_{r=1}^{m} N_r$, $\mu = \sum_{r=1}^{n} M_r$

$\xi = \sum_{r=m+1}^{q} Q_r$, $\psi = \sum_{r=n+1}^{p} R_r$

$$= (2\pi)^{\frac{2m+2n-p-q}{2}} (2\pi)^{-\left(\frac{\eta+\mu+\xi+\psi}{2}\right)} \frac{\prod_{j=1}^{m} (N_j)^{-1/2} \prod_{j=1}^{m} (N_j)^{\frac{b_j}{N_j}} \prod_{j=1}^{n} (M_j)^{-1/2} \prod_{j=1}^{n} (M_j)^{\frac{1-a_j}{n_j}}}{\prod_{j=m+1}^{q} (N_j)^{\frac{1-b_j}{N_j}} \prod_{j=m+1}^{q} (N_j)^{-1/2} \prod_{j=n+1}^{p} (M_j)^{\frac{a_j}{M_j}} \prod_{j=m+1}^{q} (M_j)^{-1/2}} \times$$

$$\frac{1}{2\pi i} \int \frac{\prod_{i=1}^{\eta} \Gamma(\beta_i - s) \prod_{i=1}^{\mu} \Gamma(\alpha_i + s)}{\prod_{i=m+1}^{\xi} \Gamma(\lambda_i + s) \prod_{i=n+1}^{\psi} \Gamma(\delta_i - s)} \left[\frac{\prod_{j=1}^{m} (N_j)^{-1} \prod_{j=1}^{m} M_j}{\prod_{j=m+1}^{q} (N_j) \prod_{j=m+1}^{p} (M_j)^{-1}} . z \right]^s ds$$

$$= (2\pi)^{\frac{2m+2n-p-q}{2}} \quad (2\pi)^{-\left(\frac{\eta+\mu+\xi+\psi}{2}\right)} \frac{\prod_{j=1}^{m} (N_j)^{-1/2} \prod_{j=1}^{m} (N_j)^{\frac{b_j}{N_j}} \prod_{j=1}^{n} (M_j)^{-1/2} \prod_{j=1}^{n} (M_j)^{\frac{1-a_j}{n_j}}}{\prod_{j=m+1}^{q} (N_j)^{\frac{1-b_j}{N_j}} \prod_{j=m+1}^{q} (N_j)^{-1/2} \prod_{j=n+1}^{p} (M_j)^{\frac{a_j}{M_j}} \prod_{j=m+1}^{q} (M_j)^{-1/2}}$$

$$G_{\xi,\psi}^{\eta,\mu} \left[\rho \left| \begin{matrix} \alpha_1, \alpha_2, \dots \dots \alpha_\mu, \delta_{n+1}, \dots \dots . . \delta_\psi, \\ \beta_1, \beta_2, \dots \dots \dots \dots \dots \beta_\eta, \lambda_{m+1}, \dots \dots \lambda_\xi \end{matrix} \right. \right] \quad \dots(2.11)$$

III. SPECIAL CASES

If we put r = 1 in I-function, then it reduces to I_H - function

$$I_{p_i,q_i:r}^{m,n}[z] = I_{p_i,q_i:r}^{m,n} \left[z \left| \begin{matrix} (q_i, \alpha_i)_{1,n} \dots . . & (q_{j1}, \alpha_{j1})_{n+1,p} \\ (b_j, \beta_j)_{1,m} \dots . . & (b_{j1}, \beta_{j1})_{m+1,q} \end{matrix} \right. \right]$$

$$= \frac{1}{2\pi i} \int_L t(s) z^s ds \quad \dots(3.1)$$

Where

$$t(s) = \frac{\prod_{j=1}^{m} \Gamma(b_j - \beta_j s) \Gamma(1 - a_j + \alpha_j s)}{\sum_{i=1}^{r} \left\{ \prod_{j=m+1}^{q_i} \Gamma(1 - b_{j1} + \beta_{j1} s) \prod_{j=n+1}^{p_i} (a_{j1} - \alpha_{j1} s) \right\}} \quad \dots(3.2)$$

put r = 1in (3.2) we get

$$I^{m,n}_{p_i,q_i:1}: 1\ [z] \qquad = I^{m,n}_{p_i,q_i:1}\left[z \left| \begin{matrix} (q_i,\alpha_i)_{1,n} \ldots\ldots & (q_j,\alpha_j)_{n+1,p} \\ (b_j,\beta_j)_{1,m} \ldots\ldots & (b_j,\beta_j)_{m+1,q} \end{matrix} \right. \right]$$

$$= \frac{1}{2\pi i} \int_L \frac{\prod_{j=1}^{m} \Gamma(b_j-\beta_j s)\ \prod_{j=1}^{n} \Gamma(1-a_j+\alpha_j s)}{\sum_{i=1}^{1}\{\prod_{j=m+1}^{q_i} \Gamma(1-b_{j1}+\beta_{j1} s)\ \prod_{j=n+1}^{p_i} \Gamma(q_{j1}-\alpha_{j1} s)\}}\ z^s ds$$

$$= \frac{1}{2\pi i} \int_L \frac{\prod_{j=1}^{m} \Gamma(b_j-\beta_j s)\ \prod_{j=1}^{n} \Gamma(1-a_j+\alpha_j s)}{\prod_{j=m+1}^{q_i} \Gamma(1-b_j+\beta_j s)\ \prod_{j=n+1}^{q_i} \Gamma(a_j-\alpha_j) s}\ z^s ds \qquad \ldots(3.3)$$

Put $\alpha_j = M_j$, $\beta_j = N_j$

$$= \frac{1}{2\pi i} \int_L \frac{\prod_{j=1}^{m} \Gamma\left(N_j\left(\frac{b_j}{N_j}-s\right)\right) \prod_{j=1}^{n} \Gamma\left(M_j\left(\frac{1-q_j}{M_j}+s\right)\right)}{\prod_{j=m+1}^{q} \Gamma\left(N_j\left(\frac{1-b_j}{N_j}+s\right)\right) \prod_{j=n+1}^{p} \Gamma\left(M_j\left(\frac{q_j}{M_j}-s\right)\right)}\ z^s ds$$

...(3.4)

Put $\frac{b_j}{N_j}$ -s = Z_j, $\frac{1-a_j}{M_j} + s = \gamma_j$, $\frac{1-b_j}{N_j} + s = Y_j$, $\frac{a_j}{M_j} - s = X_j$ in (2.4),

we get

$$= \frac{1}{2\pi i} \int_L \frac{\prod_{j=1}^{m} \Gamma(N_j Z_j)\ \prod_{j=1}^{n} \Gamma(M_j Y_j)}{\prod_{j=m+1}^{q} \Gamma(N_j Y_j)\ \prod_{j=n+1}^{p} \Gamma(M_j X_j)}\ z^s ds \qquad \ldots (3.5)$$

By using formula,[7 , p-66]

$$\Gamma(mZ) = (2\pi)^{\frac{1-m}{2}}\ m^{z-1/2}\ \prod_{j=1}^{m}\left(Z+\frac{j-1}{m}\right),$$

$$Z \neq 0, -1/m, 2/m \ldots\ldots m = 1, 2, 3 \qquad \ldots(3.6)$$

Then,

$$\prod_{j=1}^{m} \Gamma(N_j Z_j) = \Gamma(N_1 Z_1)\Gamma(N_2 Z_2)\Gamma(N_3 Z_3) \ldots\ldots\ldots \Gamma(N_m Z_m)$$

$$= \left[(2\pi)^{\frac{1-N_1}{2}} (N_1)^{Z_1-1/2} \prod_{j=1}^{N_1} \Gamma\left(Z_1+\frac{j-1}{N_1}\right)\right]$$

$$\left[(2\pi)^{\frac{1-N_2}{2}} (N_2)^{Z_2-1/2} \prod_{j=1}^{N_2} \Gamma\left(Z_2+\frac{j-1}{N_2}\right)\right]$$

$$\left[(2\pi)^{\frac{1-N_3}{2}} (N_3)^{Z_3-1/2} \prod_{j=1}^{N_3} \Gamma\left(Z_3+\frac{j-1}{N_3}\right)\right] \ldots\ldots\ldots$$

$$\left[(2\pi)^{\frac{1-N_m}{2}} (N_m)^{Z_m-1/2} \prod_{j=1}^{N_m} \Gamma\left(Z_m+\frac{j-1}{N_m}\right)\right]$$

$$= (2\pi)^{(1/2+1/2+1/2 \ldots\ldots\ldots \text{mtimes})}.\ (2\pi)^{-\left[\frac{N_1+N_2+N_3+\cdots\ldots+N_m}{2}\right]}.\quad (N_1.N_2.N_3 \ldots\ldots N_m)^{-1/2}$$

$$\prod_{j=1}^{m}(N_j)^{Z_j}.\ \prod_{j=1}^{N_1} \Gamma\left(\frac{b_1}{N_1}-s+\frac{j-1}{N_1}\right).\ \prod_{j=1}^{N_2} \Gamma\left(\frac{b_2}{N_2}-s+\frac{j-1}{N_2}\right).$$

$$\prod_{j=1}^{N_3} \Gamma\left(\frac{b_3}{N_3}-s+\frac{j-1}{N_3}\right) \ldots \prod_{j=1}^{N_m} \Gamma\left(\frac{b_m}{N_m}-s+\frac{j-1}{N_m}\right)$$

$$= (2\pi)^{m/2}.\ (2\pi)^{-\left[\frac{N_1+N_2+N_3+\cdots+N_m}{2}\right]}.\ (N_1.N_2 \ldots N_m)^{-1/2}.$$

$$N_1^{\left(\frac{b_1}{N_1}-s\right)}.N_2^{\left(\frac{b_2}{N_2}-s\right)}.N_3^{\left(\frac{b_3}{N_3}-s\right)} \ldots N_m^{\left(\frac{b_m}{N_m}-s\right)}.\prod_{j=1}^{N_1} \Gamma\left(\frac{b_1+j-1}{N_1}-s\right).$$

$$\prod_{j=1}^{N_2} \Gamma\left(\frac{b_2+j-1}{N_2}-s\right).\prod_{j=1}^{N_3} \Gamma\left(\frac{b_3+j-1}{N_3}-s\right) \ldots\ldots \prod_{j=1}^{N_m}\left(\frac{b_m+j-1}{N_m}-s\right).$$

$$= (2\pi)^{m/2}.(2\pi)^{\frac{-(N_1+N_2+N_3+\ldots\ldots+N_m)}{2}}.\ (N_1.N_2.N_3 \ldots N_m)^{-1/2}\ (N_1.N_2.N_3.\ldots\ldots N_m)^{-s}$$

$$N_1^{\frac{b_1}{N_1}}.N_2^{\frac{b_2}{N_2}}N_3^{\frac{b_3}{N_3}}\ldots\ldots\ldots\ldots N_m^{\frac{b_m}{N_m}}.\prod_{j=1}^{N_1} \Gamma\left(\frac{b_1+j-1}{N_1}-s\right)$$

$$\prod_{j=1}^{N_2} \Gamma\left(\frac{b_2+j-1}{N_2}-s\right).\prod_{j=1}^{N_3} \Gamma\left(\frac{b_3+j-1}{N_3}-s\right) \ldots\ldots \prod_{j=1}^{N_m}\left(\frac{b_m+j-1}{N_m}-s\right)$$

$$= (2\pi)^{m/2}.(2\pi)^{\frac{-(N_1+N_2+N_3+\ldots\ldots+N_m)}{2}}.\ (N_1.N_2.N_3 \ldots N_m)^{-1/2}$$

$$\prod_{j=1}^{m}(N_j)^{-s}\ N_j^{\frac{b_j}{N_j}} \prod_{i=1}^{N} \Gamma(\beta_i - s) \qquad \ldots(3.7)$$

Where $\beta_i = \frac{b_i+j-1}{N_i}$

$$N = N_1 + N_2 + N_3 + \cdots \ldots + N_m, j = 1 \text{to} N.$$

$$\prod_{j=1}^{n} \Gamma(M_j\gamma_j) \qquad = \Gamma(M_1\gamma_1)\Gamma(M_2\gamma_2).\Gamma(M_3\gamma_3) \ldots\ldots \Gamma(M_n\gamma_n)$$

$$=\left[(2\pi)^{\frac{1-M_1}{2}}(M_1)^{\gamma_1-1/2}\prod_{j=1}^{M_1}\Gamma\left(\gamma_1+\frac{j-1}{M_1}\right)\right].$$

$$\left[(2\pi)^{\frac{1-M_2}{2}}M_2^{\gamma_2-1/2}\prod_{j=1}^{M_2}\Gamma\left(\gamma_2+\frac{j-1}{M_2}\right)\right].$$

$$\left[(2\pi)^{\frac{1-M_3}{2}}(M_3)^{\gamma_3-1/2}\prod_{j=1}^{M_3}\left(\gamma_3+\frac{j-1}{M_3}\right)\right]\ldots\ldots\ldots$$

$$\left[(2\pi)^{\frac{1-M_n}{2}}(M_n)^{\gamma_n-1/2}\prod_{j=1}^{M_n}\left(\gamma_n+\frac{j-1}{M_n}\right)\right]$$

$$=(2\pi)^{(1/2+1/2+1/2\ldots\ldots\text{ntimes})}.(2\pi)^{-\left[\frac{M_1+M_2+M_3+\cdots.+M_n}{2}\right]}(M_1.M_2.M_3\ldots..M_n)^{-1/2}$$

$$\left[M_1^{\frac{1-a_1}{M_1}}.M_2^{\frac{1-a_2}{M_2}}\ldots.M_n^{\frac{1-a_n}{M_n}}\right](M_1.M_2.M_3\ldots..M_n)^{+s}.\prod_{j=1}^{M_1}\Gamma\left(\frac{1-a_1}{M_1}+s+\frac{j-1}{M_1}\right).\ \prod_{j=1}^{M_2}\Gamma\left(\frac{1-a_2}{M_2}+s+\frac{j-1}{M_2}\right).$$

$$\prod_{j=1}^{M_3}\Gamma\left(\frac{1-a_3}{M_3}+s+\frac{j-1}{M_3}\right)\ldots\ldots\prod_{j=1}^{M_n}\Gamma\left(\frac{1-a_n}{M_n}+s+\frac{j-1}{M_n}\right).$$

$$=(2\pi)^{n/2}(2\pi)^{-\left[\frac{M_1+M_2+M_3+\cdots.+M_n}{2}\right]}(M_1.M_2\ldots M_n)^{-1/2}\prod_{j=1}^{n}(M_j)^{\frac{1-a_j}{M_j}}(M_j)^{s}.$$

$$\prod_{j=1}^{M_1}\Gamma\left(\frac{j-a_1}{M_1}+s\right).\prod_{j=1}^{M_2}\Gamma\left(\frac{j-a_2}{M_2}+s\right).\prod_{j=1}^{M3}\Gamma\left(\frac{j-a_3}{M_3}+s\right)\ldots\ldots\prod_{j=1}^{M_n}\Gamma\left(\frac{j-a_n}{M_n}+s\right)$$

$$=(2\pi)^{n/2}(2\pi)^{-\frac{[M_1+M_2+\ldots.M_n]}{2}}.(M_1.M_2\ldots M_n)^{-1/2}\prod_{j=1}^{n}(M_j)^{\frac{1-a_j}{M_j}}(M_j)^{s}$$

$$\prod_{i=1}^{M}\Gamma(\alpha_i+s) \qquad \ldots(3.8)$$

Where $\alpha_i=\frac{j-a_i}{M_i}+s$

$$M=M_1+M_2+M_3+\cdots.+M_n$$

$$\prod_{j=M+1}^{q}\Gamma(N_jY_j)=\Gamma(N_{m+1}Y_{m+1}).\Gamma(N_{m+2}Y_{m+2}).\Gamma(N_{m+3}Y_{m+3})\ldots\ldots.\Gamma(N_qY_q)$$

$$=\left[(2\pi)^{\frac{1-N_{m+1}}{2}}(N_{m+1})^{Y_{m+1}-1/2}\prod_{j=1}^{N_{m+1}}\Gamma\left(Y_{m+1}+\frac{j-1}{N_{m+1}}\right)\right].$$

$$\left[(2\pi)^{\frac{1-N_{m+2}}{2}}(N_{m+2})^{Y_{m+2}-1/2}\prod_{j=1}^{N_{m+2}}\Gamma\left(Y_{m+2}+\frac{j-1}{N_{m+2}}\right)\right].$$

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$$\left[(2\pi)^{\frac{1-N_q}{2}}(N_q)^{Y_q-1/2}\prod_{j=1}^{N_q}\Gamma\left(Y_q+\frac{j-1}{N_q}\right)\right]$$

$$=(2\pi)^{\frac{q-m}{2}}.(2\pi)^{-\frac{[N_{m+1}+N_{m+2}+\cdots+N_q]}{2}}.\prod_{j=m+1}^{q}(N_j)^{\frac{1-b_j}{N_j}}(N_j)^{s}\prod_{j=m+1}^{q}(N_j)^{-1/2}$$

$$\prod_{j=1}^{N_{m+1}}\Gamma\left(\frac{1-b_{m+1}}{N_{m+1}}+s+\frac{j-1}{N_{m+1}}\right).\prod_{j=1}^{N_{m+2}}\Gamma\left(\frac{1-b_{m+2}}{N_{m+2}}+s+\frac{j-1}{N_{m+2}}\right).$$

$$\prod_{j=1}^{N_{m+3}}\Gamma\left(\frac{1-b_{m+3}}{N_{m+3}}+s+\frac{j-1}{N_{m+3}}\right)\ldots\ldots\ldots\ldots\ldots\prod_{j=1}^{N_q}\Gamma\left(\frac{1-b_q}{N_q}+s+\frac{j-1}{N_q}\right)$$

$$=(2\pi)^{\frac{q-m}{2}}(2\pi)^{-\frac{[N_{m+1}+N_{m+2}+\cdots.+N_q]}{2}}.\prod_{j=m+1}^{q}(N_j)^{\frac{1-b_j}{N_j}}(N_j)^{s}\prod_{j=m+1}^{q}(N_j)^{-1/2}.$$

$$\prod_{j=1}^{N_{m+1}}\Gamma\left(\frac{j-b_{m+1}}{N_{m+1}}+s\right)\prod_{j=1}^{N_{m+2}}\Gamma\left(\frac{j-b_{m+2}}{N_{m+2}}+s\right)\ldots\ldots\ldots\prod_{j=1}^{N_q}\Gamma\left(\frac{j-b_q}{N_q}+s\right)$$

$$=(2\pi)^{\frac{q-m}{2}}(2\pi)^{-\frac{[N_{m+1}+N_{m+2}+\cdots.+N_q]}{2}}.\prod_{j=m+1}^{q}(N_j)^{\frac{1-b_j}{N_q}}(N_j)^{s}\prod_{j=m+1}^{q}(N_j)^{-1/2}\prod_{i=m+1}^{Q}\Gamma\left(\frac{j-b_i}{N_i}+s\right)$$

$$=(2\pi)^{\frac{q-m}{2}}(2\pi)^{-\frac{[N_{m+1}+N_{m+2}+\cdots N_q]}{2}}.\prod_{j=m+1}^{q}(N_j)^{\frac{1-b_j}{N_a}}(N_j)^{s}\prod_{j=m+1}^{q}(N_j)^{-1/2}$$

$$\prod_{i=m+1}^{Q}\Gamma(\lambda_i+s) \qquad \ldots(3.9)$$

Where

$$Q=N_{m+1}+N_{m+1}+N_{m+3}+\cdots+N_q$$

$$\lambda_i=\Gamma\left(\frac{j-b_i}{N_i}+s\right)$$

$$\prod_{j=n+1}^{P}\Gamma(M_jX_j)=\Gamma(M_{n+1}X_{n+1})\,\Gamma(M_{n+2}X_{n+2})\,\Gamma(M_{n+3}X_{n+3})\ldots..\Gamma(M_PX_P)$$

$$=\left[(2\pi)^{\frac{1-M_{n+1}}{2}}(M_{n+1})^{X_{n+1}-1/2}\prod_{j=1}^{M_{n+1}}\Gamma\left(X_{n+1}+\frac{j-1}{M_{n+1}}\right)\right].$$

$$\left[(2\pi)^{\frac{1-M_{n+2}}{2}}(M_{n+2})^{X_{n+2}-1/2}\prod_{j=1}^{M_{n+2}}\Gamma\left(X_{n+2}+\frac{j-1}{M_{n+2}}\right)\right].$$
$$\left[(2\pi)^{\frac{1-M_{n+3}}{2}}(M_{n+3})^{X_{n+3}-1/2}\prod_{j=1}^{M_{n+3}}\Gamma\left(X_{n+3}+\frac{j-1}{M_{n+3}}\right)\right]\ldots\ldots\ldots$$
$$\ldots\ldots.\left[(2\pi)^{\frac{1-M_p}{2}}(M_p)^{X_p-1/2}\prod_{j=1}^{M_p}\Gamma\left(X_p+\frac{j-1}{M_p}\right)\right]$$
$$=(2\pi)^{\frac{p-n}{2}}(2\pi)^{-\frac{[M_{n+1}+M_{n+2}+\cdots+M_p]}{2}}\prod_{j=n+1}^{p}(M_j)^{\frac{a_j}{M_j}}(M_j)^{-s}\prod_{j=n+1}^{R}(M_j)^{-1/2}\prod_{i=n+1}^{p}\Gamma\left(\frac{a_i+j-1}{M_i}-s\right)$$

Where $R=M_{n+1}+M_{n+2}+\cdots+M_p$

$\delta_i=\frac{a_i+j-1}{M_i}$

$$=(2\pi)^{\frac{p-n}{2}}(2\pi)^{-\frac{[M_{n+1}+M_{n+2}+\cdots+M_p]}{2}}.\prod_{j=n+1}^{p}(m_j)^{\frac{a_j}{M_j}}(M_j)^{-s}\quad\prod_{j=m+1}^{p}(M_j)^{-1/2}\prod_{i=n+1}^{R}\Gamma(\delta_i-s)\ \ldots(3.10)$$

Putting in equation (3.3) we get

$$I_{p,q;1}^{m,n}[z]=\int_L(2\pi)^{m/2}(2\pi)^{-\frac{[N_1+N_2+\ldots+N_m]}{2}}\times\prod_{j=1}^{m}(N_j)^{-1/2}.\prod_{j=1}^{m}(N_j)^{\frac{b_j}{N_j}}\quad(2\pi)^{n/2}(2\pi)^{-\frac{[M_1+M_2+\ldots+M_n]}{2}}\prod_{j=1}^{n}(M_j)^{-1/2}$$
$$\times\frac{\prod_{j=1}^{n}(M_j)^{\frac{1-a_j}{n_j}}\prod_{i=1}^{N}\Gamma(\beta_i-s)\prod_{i=1}^{M}\Gamma(\alpha_i+s)\prod_{j=1}^{m}(N_j)^{-s}\prod_{j=1}^{n}(M_j)^{s}}{(2\pi)^{\frac{q-m}{2}}(2\pi)^{-\frac{[N_{m+1}+N_{m+2}+\cdots+N_q]}{2}}\prod_{j=m+1}^{q}(N_j)^{\frac{1-b_j}{N_j}}\prod_{j=m+1}^{q}(N_j)^{-1/2}(2\pi)^{\frac{p-n}{2}}}$$
$$\times\frac{1}{(2\pi)^{-\frac{[M_{n+1}+M_{n+2}+..+M_p]}{2}}\prod_{j=m+1}^{p}(M_j)^{\frac{a_j}{M_j}}\prod_{j=m+1}^{q}(M_j)^{-1/2}\prod_{i=m+1}^{Q}\Gamma(\lambda_i+s)}$$
$$\times\frac{1}{\prod_{i=n+1}^{R}\Gamma(\delta_i-s)\prod_{j=m+1}^{q}(N_j)^{s}\prod_{j=n+1}^{p}(M_j)^{-s}}z^s ds$$
$$=(2\pi)^{\frac{m+n-q+m-p+n}{2}}\quad(2\pi)^{-\frac{[(N_1+N_2+\ldots+N_m)+(M_1+M_2+\cdots M_n)+(N_{m+1}+N_{m+2}+\cdots+N_q)+(M_{n+1}+M_{n+2}+\cdots+M_p)]}{2}}$$
$$\times\frac{\prod_{j=1}^{m}(N_j)^{-1/2}\prod_{j=1}^{m}(N_j)^{\frac{b_j}{N_j}}\prod_{j=1}^{n}(M_j)^{-1/2}\prod_{j=1}^{n}(M_j)^{\frac{1-a_j}{n_j}}.}{\prod_{j=m+1}^{q}(N_j)^{\frac{1-b_j}{N_j}}\prod_{j=m+1}^{q}(N_j)^{-1/2}\prod_{j=m+1}^{p}(M_j)^{\frac{a_j}{M_j}}\prod_{j=m+1}^{q}(M_j)^{-1/2}}$$
$$\times\int_L\frac{\prod_{i=1}^{N}\Gamma(\beta_i-s)\prod_{i=1}^{M}\Gamma(\alpha_i+s)\prod_{j=1}^{m}(N_j)^{-s}\prod_{j=1}^{n}(M_j)^{s}}{\prod_{i=m+1}^{Q}\Gamma(\lambda_i+s)\prod_{i=n+1}^{R}\Gamma(\delta_i-s)\prod_{j=m+1}^{q}(N_j)^{s}\prod_{j=n+1}^{p}(M_j)^{-s}}z^s ds$$

Where $\eta=\sum_{r=1}^{m}N_r,\quad\mu=\sum_{r=1}^{n}M_r$

$\xi=\sum_{r=m+1}^{q}Q_r,\ \ \psi=\sum_{r=n+1}^{p}R_r$

$$=(2\pi)^{\frac{2m+2n-p-q}{2}}(2\pi)^{-\left(\frac{\eta+\mu+\xi+\psi}{2}\right)}\frac{\prod_{j=1}^{m}(N_j)^{-1/2}\prod_{j=1}^{m}(N_j)^{\frac{b_j}{N_j}}\prod_{j=1}^{n}(M_j)^{-1/2}\prod_{j=1}^{n}(M_j)^{\frac{1-a_j}{n_j}}}{\prod_{j=m+1}^{q}(N_j)^{\frac{1-b_j}{N_j}}\prod_{j=m+1}^{q}(N_j)^{-1/2}\prod_{j=n+1}^{p}(M_j)^{\frac{a_j}{M_j}}\prod_{j=m+1}^{q}(M_j)^{-1/2}}\times$$
$$\frac{1}{2\pi i}\int\frac{\prod_{i=1}^{\eta}\Gamma(\beta_i-s)\prod_{i=1}^{\mu}\Gamma(\alpha_i+s)}{\prod_{i=m+1}^{\xi}\Gamma(\lambda_i+s)\prod_{i=n+1}^{\psi}\Gamma(\delta_i-s)}\left[\frac{\prod_{j=1}^{m}(N_j)^{-1}\prod_{j=1}^{m}M_j}{\prod_{j=m+1}^{q}(N_j)\prod_{j=m+1}^{p}(M_j)^{-1}}.z\right]^s ds$$
$$=(2\pi)^{\frac{2m+2n-p-q}{2}}\quad(2\pi)^{-\left(\frac{\eta+\mu+\xi+\psi}{2}\right)}\frac{\prod_{j=1}^{m}(N_j)^{-1/2}\prod_{j=1}^{m}(N_j)^{\frac{b_j}{N_j}}\prod_{j=1}^{n}(M_j)^{-1/2}\prod_{j=1}^{n}(M_j)^{\frac{1-a_j}{n_j}}}{\prod_{j=m+1}^{q}(N_j)^{\frac{1-b_j}{N_j}}\prod_{j=m+1}^{q}(N_j)^{-1/2}\prod_{j=n+1}^{p}(M_j)^{\frac{a_j}{M_j}}\prod_{j=m+1}^{q}(M_j)^{-1/2}}$$
$$G_{\xi,\psi}^{\eta,\mu}\left[\rho\left|\begin{matrix}\alpha_1,\alpha_2,\ldots\ldots\alpha_\mu,\delta_{n+1},\ldots\ldots..\delta_\psi,\\ \beta_1,\beta_2,\ldots\ldots\ldots\ldots\ldots\beta_\eta,\lambda_{m+1},\ldots\ldots\lambda_\xi\end{matrix}\right.\right]\qquad\ldots(3.11)$$

The transformation relations derived above can generate new properties of the I-function and other higher order generalized Hypergeometric functions.

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